Application of Teaching Methods in Mathematics at Secondary Level in Pakistan

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Abstract
The purpose of this study is to discuss the teaching methods and their application in different branches of mathematics taught at secondary level in Pakistan. Teaching methods of mathematics include lecture, inductive, deductive, heuristic or discovery, analytic, synthetic, problem solving, laboratory and project methods. Teachers may adopt any method according to the specific unit of syllabus, available resources and number of students in a class. Different merits and demerits of teaching methods along with the relevance of each method to the appropriate branches of mathematics in Pakistani context are explained in this paper.

Keywords: Mathematics; Pedagogy; Teaching Methods; Secondary Level

I. Introduction
The word mathematics came from a Greek word “μάθημα” (máthēma) which means science or study. Mathematics is "the branch of human enquiry involving the study of numbers, quantities, data, shape and space and their relationships, especially their generalizations and abstractions and their application to situations in the real world" (Clapham & Nicholson, 2009, p. 505). Mathematicians generalise new formulas or methods based on similar patterns for different branches of mathematics (Devlin, 2004). Before teaching mathematics, every teacher should be informed well about the educational values of this subject. Proper teaching method should also be adopted according to the situation, learning environment and educational background of the students. It is very important to keep the motivational level of students high otherwise they lose interest in mathematics (Butler & Wren, 1965). Students can be motivated by highlighting the importance of this subject, for example, mathematics is quite essential to learn other science related subjects. Moreover, students can avail good employment opportunities in their future life because of diverse applicability of mathematics in many fields (Rani, 2007). Teachers should be cleared about the following goals of teaching mathematics (Cornelius, 1982; Sidhu, 1995).

i. To develop reasoning ability in thinking process of the students.
ii. To enable students to do different kinds of calculations related to the daily life problems.
iii. To make them creative by developing analytical and discovering abilities in them.
iv. To enable them to learn other subjects of science or general science.
v. To prepare them for higher studies.
vi. To develop scientific approach in them to understand the realities of life on the base of logic.
vii. To enable them to find out the similar patterns in one particular activity or phenomenon for generalising the results from them.

viii. To prepare them for all those fields of life in which mathematics is applicable.

II. Teaching Mathematics at Secondary Level in Pakistan

Curriculum of mathematics at secondary level including grade 9 and 10 in Pakistan is developed and updated by the ministry of education. All provinces of the country including Punjab, Sindh, Baluchistan and Khyber Pakhtunkhwa have their own separate textbook boards which publish textbooks following the national curriculum. Textbooks for secondary and higher secondary level are taught in all government and private schools in Pakistan except those which are following the British or American educational systems. National curriculum for mathematics at secondary level (Ministry of Education, 2002) has been divided into different small units in which the benchmarks are clearly mentioned. These units are designed for different branches of mathematics like number and operations, algebra, geometry, information handling and trigonometry etc.

Pakistan is a developing country where educational situation is not very much encouraging. Lack of resources and funds are the biggest hurdles in educational reforms. Situation in private schools is relatively good but their fee structure is very much high and not affordable by everyone. Majority of the students are studying in the public schools which are in very poor condition. Properly trained teachers are not available everywhere. Strength of students is very much high in most of the schools so that teachers cannot give attention to each student in a proper way. Pedagogy of mathematics includes application of different teaching methods like lecture, inductive, deductive, heuristic or discovery, analytic, synthetic, problem solving, laboratory and project methods. Instructional methodology of every teacher should be adaptive according to each unit of syllabus, available resources and strength of the students.

III. Lecture Method

In this method, knowledge is delivered through a speech. This is the oldest and most important teaching method because it is always remained a part of all other instructional methodologies. In this method, a teacher takes part as an active participant and students are at the receiving end most of the time. That is why; it is a teacher centred approach. This is also referred to as direct instruction, training model (Joyce, Weil, & Shower, 1992), active teaching (Good, Grouws, & Ebmeier, 1983) and explicit instruction (Rosenshine & Stevens, 1986). Lecture method is not only used for teaching theoretical concepts but it is also helpful for giving training of complex skills and procedures.

A. Merits and Demerits of Lecture Method

Lecture method has also some merits (Sidhu, 1995; Sellers, Roberts, Giovanetto, Friedrich, & Hammargren, 2007) which are as follows.

i. This is the most convenient and easy method.

ii. This is the fastest way to deliver knowledge so when the syllabus is so heavy then it becomes necessary.

iii. When strength of a class is very high then it becomes more important.

iv. This is so economical because there is no equipment involved in it and only one teacher can teach so many students.
v. This is very helpful to introduce new concepts.
vi. This can be used to raise the interest level of the students while applying any other teaching method.
vii. This method has also some demerits which are listed below (Singh, 2007; Sellers et al., 2007).
viii. This is a teacher centred approach so students cannot play an active role.
ix. This method does not develop reasoning and thinking ability in the students.
x. Sometimes lectures become boring because there is no activity involved in it.
xi. In this method, teacher-student relationship is not developed in proper way.
xii. This method is relatively more useful in higher classes.
xiii. It becomes essential to enhance writing and communication skills by the teachers.

B. Application of Lecture Method in Mathematics at Secondary Level

As no practical work is involved in this method, so it can only be used to clarify the basic concepts of each unit given in the textbooks of mathematics. It is applicable to teach all branches of mathematics including sets, logarithms, algebra, matrices, statistics, geometry and trigonometry. Mathematical problems related to these branches cannot be solved by this method but the procedures and methods to solve them can be explained in a very good manner. The historical perspective of these branches and their relevance to the real life can also be described by this method.

IV. Inductive Method

This method is also called scientific method in which we proceed from known to unknown, from specific to general and from example to rule or formula. In this method based on induction, students are presented some similar examples or problems related to one particular domain. Then students try to establish a formula, rule, law or principal by observing them. If a generalised result is true for those similar examples or problems then it would also be true for all other such kind of examples (Sidhu, 1995).

A. Merits and Demerits of Inductive Method

This method has also some merits and demerits (Neubert & Binko, 1992; Sekhar, 2006). Its merits are as follows.

i. This method is useful to introduce a new mathematical concept along with a formula or rule.
ii. Students who like the inductive approach can infer the more complicated rules or formulas (Felder, 1993).
iii. This is a student centred approach because students play an active role in it.
iv. As the students may establish laws and principles by themselves so it gives them confidence.
v. This method helps to motivate the students to think logically and make the learning environment more interesting.
vi. This is based on reasoning and experimentation.
vii. This is quite suitable for primary and secondary level classes.
viii. Students easily remember the laws or principles which they prove by themselves.

This method has some demerits as well.

i. It is quite time consuming and laborious as well.
To establish a law or principle is not the complete process of learning. Students have to practise a lot to understand the concept fully.

Sometimes a formula or rule proved with the help of some similar examples does not applicable in other similar cases.

Only experienced teachers can use this method in a right way.

This method does not help in developing problem solving ability in the students.

B. Application of Inductive Method in Mathematics at Secondary Level

Inductive method is used to establish laws, principals, formulas and methods instead of solving mathematical problems. Therefore it can be used in all branches of mathematics but establishing laws or formulas at the secondary level is only involved in algebra, matrices and to some extent geometry.

V. Deductive Method

This method is totally different from inductive method. In this method, we proceed from general to specific and from a rule to an example. Already constructed formulas, rules, methods or principles are taught to the students and they apply them to solve the problems (Sidhu, 1995). In this teaching approach, we can also prove a theorem with the help of undefined terms, defined terms, axioms and postulates. Then with the help of that theorem along with different rules and principles, we can derive other theorems as well (Singh, 2007).

A. Merits and Demerits of Deductive Method

There are some merits and demerits of this method as well (Sekhar, 2006). Some merits are listed below.

i. This method is very easy and short.

ii. To remember a formula or rule is not very difficult so this method is blessing for those students who cannot remember complicated procedures (Brigham & Matins, 1999).

iii. Teachers can complete the syllabus easily by this method.

iv. This method helps to enhance the computational ability of the students.

v. It is helpful to teach those concepts in which derivation of rules or methods is not involved.

vi. With the help of this method, we can prove different theorems using already defined formulas or principles.

This method has the following demerits.

i. It becomes very difficult for students when they have to remember so many rules and formulas.

ii. This method does not help in improving reasoning ability in the students.

iii. It is not effective at lower level classes.

iv. This method is not constructivist. If a student forgets a rule or principle then he or she cannot reconstruct that easily (Sidhu, 1995).

v. This method does not encourage discovery learning.

vi. It cannot make students creative.

vii. Students may be doubtful about the reason of using one particular formula.
B. Application of Deductive Method in Mathematics at Secondary Level

Deductive method is the highly used method in mathematics. It is used to solve those problems in which complicated procedures are not involved and they can be solved by applying different kinds of already established laws, methods, formulas and principles directly. Such kind of problems can be found in all units of syllabus of mathematics at secondary level including sets, logarithms, algebra, matrices, variation, statistics, geometry and trigonometry.

VI. Heuristic Method

The word heuristic was drawn from a Greek word “heurisko” which means “I find out”. Heuristic method is based on child’s psychology who always wants to discover something by himself or herself. That is why it is also known as discovery method (Bruner, 1960, 1962, 1966). Sometimes a teacher only focuses on delivering lectures through speech in which students do not actively participate and get bored most of the time. But in the heuristic method, students are encouraged to reach the solution by constructing the knowledge themselves. Teacher only facilitates them by raising relevant questions. That is why it is also called inquiry method (Suchman, 1962). As students discover the solution under the guidance of a teacher so it is also known as guided discovery method or programmed instruction. So many researches (Ashton, 1962; Wills, 1967; Wilson, 1967) have proved that heuristic or discovery method is more effective in teaching mathematics than expository approach.

A. Merits and Demerits of Heuristic Method

This method has merits and demerits as well (Singh, 2007; Sidhu, 1995). Its merits are as follows.

i. It is a student centred approach.
ii. It gives confidence to the students because they discover the solution by themselves.
iii. It makes students creative.
iv. It develops reasoning and thinking abilities in the students.
v. It clears concepts in a better way.
vi. Continuously inquiring the students keeps them active and they do not get bored.

Demerits of this method are given below.

i. This method is quite time consuming.
ii. It is essential for all teachers to be properly skilled with this method otherwise it is very difficult for them to apply this in the classroom.
iii. If any student has less aptitude towards discovery then it becomes very difficult for him or her to learn something through this method.
iv. It is only applicable if strength of a class is low but it is usually not possible in public schools of Pakistan.
v. If a teacher fails to give proper guidance to the students then they may get discouraged.
vi. This method is not suitable for teaching all kinds of mathematical problems.
vii. Sometimes a teacher fails to ask proper questions during the discovery process so that distracts students.
viii. With the help of this method, lengthy syllabus cannot be finished in time.
B. Application of Heuristic Method in Mathematics at Secondary Level

Heuristic method can be used to teach all branches of mathematics. It is helpful when students are not master to solve problems related to one particular concept and they need guidance. When students get master of different methods and formulas then they are encouraged for deductive or problem solving methods to solve the same problems.

VII. Analytic Method

In this method, we analyse the problem first by breaking up the problem in small segments and then move towards solution. It is also called descriptive method. It leads us from the unknown part of the problem to something already known or given in the problem statement. This method emphasises on why we are applying different kinds of operations and what is the relationship between the required solution and other portions of the problem (Rani, 2007; Singh, 2007).

A. Merits and Demerits of Analytic Method

Analytic method has also some merits and demerits (Sidhu, 1995; Sekhar, 2006). Merits of this method are as follows.

i. This is a pure logical method so there is always less chance of doubts.
ii. Discovering the solution is an essential part of this method so it enhances logical thinking and reasoning ability of the students (Agarwal, 1992).
iii. Students always play an active role in this method.
iv. Students do not need to memorise any set procedure to solve a problem.
v. It encourages scientific attitude.

This method has the following demerits.

i. This method is quite lengthy and time consuming (Agarwal, 1992).
ii. This is not suitable for all kinds of problems.
iii. Only skilled teachers can apply this method.
iv. This is not suitable if the syllabus is so lengthy.

B. Application of Analytic Method in Mathematics at Secondary Level

Because of discovery approach, only such kind of problems can be taught with the help of this method in which we have to prove something. At secondary level, such problems can only be found in the units of algebra, geometry, ratio and proportion (variation).

VIII. Synthetic Method

This method is completely opposite to the analytic method as we proceed from the given or known elements in the problems to the desired solution or unknown. In this method, we synthesise or put together separate elements or small portions given in the problems to draw a series of conclusions until the unknown or desired result is found (Sidhu, 1995). This method is quite simple and led by analytic method. Process of analysis in analytic method clears the basics of any concept. On the other hand, synthetic method is based on already learnt concepts. Therefore it is quite necessary to go through the analytic method to become master of specific mathematical concepts then synthetic method can be used to solve the problems more quickly. In this method, students are not
bound to give reason for each and every step while solving a mathematical problem. That is why it cannot be preferred alone to derive mathematical proofs (Butler & Wren, 1965).

A. Merits and Demerits of Synthetic Method

This method has also some merits (Agarwal, 1992; Sekhar, 2006) as given below.

i. Synthetic method is short and brief.
ii. It is quick because of deductive reasoning.
iii. It sharpens the memory of students.
iv. Teachers can finish the lengthy course in time through it.
v. It provides opportunity to the students to practise mathematical formulas or procedures.

Demerits of this method are as follows (Sidhu, 1995; Singh, 2007).

i. It is not student centred.
ii. It does not develop reasoning ability in the students.
iii. Students have to remember so many steps without reasoning.
iv. It does not employ heuristic approach.
v. If a student forgets any mathematical proof then it is very difficult to recall it step by step.
vi. It does not clarify the concepts completely.
vii. It is neither psychological nor scientific in nature.

B. Application of Synthetic Method in Mathematics at Secondary Level

Just like analytic method, this method can be used for such problems in which we have to prove something. It is also useful to find out something unknown with the help of given conditions in the problem statement. These problems can be found in the units of algebra, ratio and proportion (variation) and geometry at secondary level.

IX. Problem Solving Method

Instructional methodologies should improve reasoning ability in the students. In this way, they become capable to find out the solutions of different kinds of problems not only during the studies but in their daily routine matters as well. Every child has the curiosity to explore the things and this psychology of the children can be utilised in a better way through problem solving method. It is the most important instructional methodology for mathematics (Collier & Lerch, 1969). Bruner, Oliver, Greenfield (1966) and Gagné (1970), the most famous psychologists, also gave the top priority to this method.

In this method, students are given such problems which cannot be solved easily or their solutions are not obvious. A student tries to reach the goals or solutions through the set of events or procedures. Gagné (1970) calls these events or procedures as lower order capabilities in which formulas, rules and concepts are used from which a student is already familiar. According to him, what the student learns is called a higher order principle which is the result of lower order capabilities.
A. Merits and Demerits of Problem Solving Method

Problem solving method has also some merits and demerits. There are the following merits of this method (Taplin, 1995; Singh, 2007).

i. This method is scientific in nature.
ii. It is student centred.
iii. It is helpful to enhance the reasoning ability of the students.
iv. Students are provided opportunity to apply their previous knowledge through problem solving.
v. Students learn how to face totally new situation by solving different kinds of questions.
vi. Teacher can assess the abilities of his or her students easily.
vii. This method improves logical thinking in the students which leads towards creativity.

There are some demerits of this method as well (Sidhu, 1995; Singh, 2007).

i. This method is quite time consuming.
ii. This is usually not recommended for lower classes.
iii. Textbooks do not provide enough help to apply this method because such books are usually written in a traditional way.
iv. Logical thinking is involved in this method therefore physical kind of activities are totally neglected.

B. Application of Problem Solving Method in Mathematics at Secondary Level

This method is used to solve those complicated problems which cannot be solved with the help of single law or formula. Usually word problems are solved with it. At secondary level, such kind of problems can be found in the units of algebra, trigonometry, ratio and proportion (variation).

X. Laboratory Method

Mathematics is different from the subjects involving readings thus practical work is its major part. Laboratory method has the capacity to deal with practical work in mathematics. It is a method of “learning by doing”. That is why, different kinds of tools and equipments are used in it to perform practical work which includes drawing of different shapes, taking measurements of geometrical figures and making of charts and graphs. Students go through different experiments in laboratory or classroom and learn by observing and calculating themselves. During this process, they get opportunity to draw conclusions and generalise different laws and formulas. Therefore, this method can be said an extended form of inductive method (Sidhu, 1995).

The role of a teacher in this method is to supervise the whole process and give proper instructions to the students at each step. He or she should keep some points in mind to make this method successful (Singh, 2007).

i. Necessary equipments related to the laboratory work should be arranged in advance.
ii. Teacher should continuously observe the practical work of every student and guide him or her accordingly.
iii. Every student should be encouraged throughout the practical work.
iv. All necessary concepts should be cleared before starting experimental work.

If number of the students is high and required equipment is not enough then students can be divided into small groups.

A. Merits and Demerits of Laboratory Method

This method has also some merits and demerits (Sekhar, 2006). Merits of this method are as follows.

i. It is student centred method.
ii. Students play an active role so they do not get bored.
iii. It is based on discovery approach.
iv. Knowledge gained through practical work is long lasting.
v. As students establish laws and formulas by themselves so they gain confidence.
vi. Practical utilisation of mathematics is realised by the students.
vii. When students work in the groups then their learning becomes fast because of sharing information and ideas.
viii. The teacher-student relationship gets strengthened.

Laboratory method has the following demerits.

i. It is very lengthy process.
ii. It is restricted to those topics only in which practical work is involved.
iii. In Pakistan, it is very difficult for so many schools to spend a lot of money on tools and equipments involved in this method.
iv. Teachers have to practise a lot before applying this method in the classroom or laboratory.
v. Students cannot practise this method to establish laws or principles independently.
vi. It is more effective in lower level classes as compare to secondary level.

B. Application of Laboratory Method in Mathematics at Secondary Level

This method is mostly used for practical geometry. At the secondary level, it can also be used to establish or verify the laws and theorems in sets and trigonometry. These laws and theorems are usually proved through inductive method but laboratory method can be used at alternative basis to create interest among the students.

XI. Project Method

This method is also based on the philosophy of “learning by doing”. It was devised by famous educationist Prof. Dr. William H. Kilpatrick who defined this method as “whole-hearted purposeful activity” (Kilpatrick, 1918). In this method, students are engaged in such kind of projects in which they get opportunity to apply their theoretical knowledge and learn practically. In these projects, students work in natural environment outside or within the boundary of school. During this process, they face different mathematical kind of problems in real life and then try to solve them with previously gained knowledge. Projects may be allocated at individual level but usually students are divided in the small groups to accomplish them (Sidhu, 1995).
Project method provides cooperative learning in which not only students share the ideas and knowledge but they also get motivated to complete the tasks as soon as possible. Famous educationist John Dewey (1916) emphasised on social interaction of the learners for the first time then Herbert Thelen (1954, 1960) also gave importance to cooperative learning in small groups.

A. Merits and Demerits of Project Method

There are some merits and demerits of this method (Sekhar, 2006). Its merits are as follows.

i. It is totally student centred method.

ii. It helps students to correlate the mathematical knowledge with real life problems.

iii. It is a social activity that helps to promote friendly environment among students.

iv. Students share their ideas and experiences with each other.

v. It gives confidence to the students.

vi. Students learn so many other things during projects in real life scenarios.

vii. Students remain active and enjoy throughout the project.

Project method has the following demerits.

i. It is quite time consuming.

ii. It is costly because so many equipments are involved in it.

iii. Because of excessive practical work, students cannot give much attention to practise the mathematical operations.

iv. Usually textbooks are not designed according to this method.

v. It is very difficult to complete the syllabus in time with the help of this method especially when strength of a class is very high.

B. Application of Project Method in Mathematics at Secondary Level

This method is not used to teach one particular concept of mathematics. When students get master of different areas of mathematics like algebra, geometry or trigonometry with the help of other teaching methods then project method provides opportunity to them to apply their already learnt knowledge in real life scenarios.

XII. Conclusion

This study described different teaching methods of mathematics at secondary level in Pakistani context. These teaching methods include lecture, inductive, deductive, heuristic, analytic, synthetic, problem solving, laboratory and project methods.

Lecture method can be used to explain basic concepts of all branches of mathematics. Inductive method is helpful to establish laws and formulas related to algebra, matrices and geometry. Already established laws and formulas can be applied through deductive method to solve problems related to all branches of mathematics. If students have not proper command to solve problems then heuristic or discovery method can be applied in which inquiry approach is quite helpful to make students capable to understand mathematical procedures. It is quite time consuming but it enhances reasoning ability in the students. The mathematical problems in which students have to prove laws or formulas can be taught with the help of analytic method. Such kinds of problems can be found in the units of algebra, ratio and proportion (variation) and geometry. When
students become master to analyse the problems then they are in the position to synthesise and reach the goal more quickly using already learnt concepts. The problems in which something has to be proved can also be taught through synthetic method. This method is short and brief as compared to analytic method.

There are also some lengthy problems which cannot be solved directly by applying a single formula or a small procedure then problem solving method may be adopted. Lengthy word problems can be found in the units of algebra, trigonometry, ratio and proportion (variation). To prove laws and theorems related to sets and trigonometry involves practical work. For this purpose, laboratory method can be used. Practical geometry is totally dependent on this method. As for as project method is concerned that provides opportunity to the students to relate their theoretical knowledge about mathematics with their real life scenarios. Students get involved in different small projects and they try to get solutions by applying laws and formulas of different branches of mathematics.

A teacher should be familiar with all of these teaching methods because he or she can get better results by applying appropriate method according to the nature of a problem, available resources and number of students in a class.

References


